NUCLEAR REACTOR THEORY

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Summary

Nuclear fission is a very effective source of energy. It is released when a fissile nucleus such as Uranium-235 splits into two lighter fission fragments. Fission is accompanied by the release of two or three neutrons. If just one of these goes on to interact with another Uranium-235 nucleus causing it to fission, a non-diverging chain reaction is established.

Neutrons interact in some way with the nuclei of all materials, generally being absorbed, so nuclear reactor materials have to be carefully selected for their nuclear properties. Furthermore the probability of a neutron causing fission in Uranium-235 varies with neutron energy, being greater at low energies. Neutrons therefore need to be slowed from their high energy of release to a suitably low energy to cause fission. This is done in a moderator which is effective in reducing neutron energy by appropriate interactions but is not a strong absorber of neutrons.

Natural uranium consists of only 0.7 percent fissile Uranium-235 with the remainder being mainly non-fissile Uranium-238. To increase the probability of fission reactions occurring in the fuel, natural uranium is usually enriched in Uranium-235 to a few percent. Generally the fuel and moderator have to be matched to ensure satisfactory neutron interactions to maintain a continuous chain reaction. Usually the fuel is lumped into rods with the moderator between the rods to ensure effective slowing down of the neutrons. The heat produced by the fission processes is removed by a coolant flowing through the reactor. Generally the coolant flows in contact the fuel rods to ensure effective removal of heat. In some reactors the coolant also serves as the moderator. Whether it does or not, it must have properties similar to that of the moderator and not absorb neutrons strongly.

The interactions of neutrons with the nuclei of various materials is quite complex. Various models to facilitate the visualization of the processes have been devised.

Though not necessarily physically correct they do promote an understanding of certain nuclear phenomena.

1. Nuclear Physics

1.1. Atomic Structure

Atoms are made up of a nucleus of *protons* and *neutrons* which is surrounded by a cloud of *electrons*. Different *elements* have different numbers of protons in the nucleus while different *isotopes* of the same element have different numbers of neutrons in the nucleus. All isotopes of all elements are commonly known as *nuclides*. Hence there is a Chart of Nuclides showing all isotopes of all elements. The number of negatively charged electrons is always equal to the number of positively charged protons. For the lighter elements the number of neutrons in the nucleus is approximately equal to the number of protons.

The masses of a proton and a neutron are exceeding low being only about 1.67×10^{-27} kg (1.67×10^{-24} g). The mass of an electron is very much less being about 0.00091×10^{-27} kg (0.00091×10^{-24} g). The electron mass is practically negligible in comparison with a proton and a neutron. The neutron has a slightly greater mass than the proton since a neutron is made up of a proton plus an electron.

1.2. Atomic Notation

The *atomic number* is designated Z. This is the number of protons in the nucleus. It also designates a particular element. The *neutron number* is designated N. This is the number of neutrons in the nucleus. Together neutrons and protons are termed *nucleons* and the *atomic mass number*, designated A, is the total number of nucleons in the nucleus. The atomic mass number is related to the *isotopic mass* (or atomic weight) of the element but is an integer whereas the isotopic mass (or atomic weight) is the actual mass of the particular isotope (or element). The usual way of designating a particular isotope X is as follows:

Isotope ${}^{A}X_{Z}$
Element = X
Mass Number = A
Protons = Z
Neutrons = $A - Z$

1.3. Atomic Mass Scale

Since atomic masses are so minute, it is convenient to introduce a very small unit, known as the *atomic mass unit* (u), to simplify the arithmetic. This unit is defined by taking the mass of the neutral atom of the isotope carbon-12 to be precisely 12 u. From this, it follows that the equivalence between the atomic mass unit and the kilogram is:

$1 \text{ u} = 1.660566 \times 10^{-27}$ kg

The masses of the atomic constituents in atomic mass units are:

Proton	1.0072765 u
Neutron	1.0086650 u
Electron	0.0005486 u

The isotopic masses of all other elements are given relative to Carbon-12. Isotopic masses for all isotopes are given in the Chart of Nuclides. The atomic weight M of an element is calculated by adding the products of the isotopic masses and the relative abundances of the individual isotopes.

$$M = \gamma_{\mathrm{A}} M_{\mathrm{A}} + \gamma_{\mathrm{B}} M_{\mathrm{A}} + \gamma_{\mathrm{C}} M_{\mathrm{C}} \ldots \qquad (1)$$

Note that the isotopic masses are the total masses including electrons. If the mass of the nucleus only is required then the masses of electrons must be subtracted. For nuclear reaction equations where the same number of electrons appear on each side of the equation the masses of electrons may be neglected since they cancel one another.

1.4. Mass-Energy Equivalence

It is well known that energy E is related to mass m according to the following equation where c is the velocity of light.

$$E = mc^2 \qquad (2)$$

From the above the mass of one atomic mass unit is:

1 u = 1.660566×10^{-27} kg

The velocity of light is:

$c = 2.998 \times 10^{8}$ m/s

The energy equivalent of one atomic mass unit may therefore be calculated as follows:

1 u = 1.4925×10^{-12} J

In nuclear physics it is common to use *electron volts* (eV) or *mega-electron volts* (MeV) as a measure of energy where:

1 eV = 1.6022×10^{-19} J

Making this substitution gives the energy equivalent of one atomic mass unit as

1 u = 931.5 MeV

1.5. Avogadro's Number

It is often required in nuclear engineering that the number of atoms or nuclei in a given sample be calculated. This number can be obtained from Avogadro's Number given as:

$N_A = 6.022 \times 10^{23}$

Note that to facilitate calculations it is better to write it immediately as:

$N_A = 0.6022 \times 10^{24}$

This renationalizes the exponent and allows it to be easily cancelled by other exponents also written using the engineering notation for exponents.

The number of atoms N in a given sample of mass m is given by the following equation where m is the mass in grams and M is the atomic weight

$$N = \left(N_A / M \right) m \quad (3)$$

In many cases since the atomic mass number A is very nearly equal to the atomic weight M of an element the following approximate relationship may be used

$$N \approx \left(N_A / A \right) m \quad (4)$$

If the sample consists of a mixture of isotopes with one predominating then the atomic mass number of the predominant isotope may be used. This approximation gives results within an acceptable range of accuracy in engineering calculations where other simplifying assumptions are made.

As an example of the use of this relationship consider the potential power output of the total consumption of 1 kg of pure Uranium-235 per day. The number of Uranium-235 atoms in 1 kg of pure fuel is:

$N = (0.6022 \times 10^{24} / 235) \times 1000$

$N = 2.562 \times 10^{24}$ atoms

If this number of atoms is totally consumed (fissioned) in one day and if each fission produces 200 MeV of energy then energy is released at the following rate:

$P = 0.005932 \times 10^{24}$ MeV/s

If this is converted to watts and megawatts the rate of heat energy production is:

$P = 950$ MW

If used in a nuclear power plant with a thermal cycle efficiency of about 30 percent this would be equal to approximately 300 MW of electrical power.

If, in the above equation, the atomic weight of Uranium-235 (235.043924) had been used instead of the atomic mass number (235) the difference in the answer would have been negligible.

1.6. Atomic Dimensions

The dimensions of atoms are extremely small and impossible to visualize. It is therefore necessary to draw a comparison and to compare the size of a complete atom with that of its nucleus.

The diameters of atoms range from about 75 picometers to about 500 picometers with the larger atoms generally being towards the bottom left hand side of the Periodic Table of Elements.

The radii and diameters of nuclei are given by the following formulae where A is the atomic mass number:

$$r = 1.25\times10^{-15} A^{1/3} \text{ m}$$

$$d = 2.5\times10^{-15} A^{1/3} \text{ m}$$

Considering Helium as an example, the following dimensions are obtained;

$$\textit{Atom diameter} = 100\times10^{-12} \text{ m}$$
$$= 100\,000 \times 10^{-15} \text{ m}$$

$$\textit{Nucleus diameter} = 0.004\times10^{-12} \text{ m}$$
$$= 4 \times 10^{-15} \text{ m}$$

The atom diameter is thus some 25 000 times that of the nucleus. It is evident that an atom consists mainly of empty space. This is an important concept when considering the passage of neutrons and other atomic particles through a material. Uncharged particles such as neutrons pass freely through millions of atoms before eventually striking or interacting with a nucleus.

2. Energy Levels

Electrons occupy various shells K, L, M, N, O, etc. and subshells s, p, d, f, etc. surrounding the nucleus. With increasing atomic number these shells are filled progressively. The maximum number of electrons which may occupy these shells is 2, 8, 18, 32 and 50. According to the Pauli exclusion principle however the subshells are filled in such a way as to obtain the lowest energy state. This means that the third and higher shells may not be filled before electrons begin to occupy subsequent shells. This is the case for elements with atomic numbers greater than 18. Potassium for example has 2, 8, 8 and 1 electrons respectively occupying shells K, L, M, and N.
A similar principle is true for neutrons and protons in the nucleus. The Pauli exclusion principle prohibits similar particles with the same spins being at the same energy level.

Thus not more than two protons or two neutrons with opposite spins may occupy any energy level but, since protons and neutrons are different particles, they may together occupy the same energy level. A set of two protons and two neutrons each with opposite spins may therefore occupy a single energy level. If however there were instead four protons or four neutrons only two could occupy the lowest energy level while the remaining two would have to occupy a higher energy level assuming that their spins were opposite. It is evident therefore that the nucleons in nuclei with equal numbers of neutrons and protons will assume the lowest energy levels. The lowest energy level represents the most stable condition and nuclear reactions will be driven naturally towards conditions of lower energy. Nuclear transformations arising from radioactive decay invariable allow the nucleons to rearrange themselves in a lower energy state and in so doing emit excess energy.

The electrons surrounding a nucleus may be excited to discrete energy levels up to the energy level at which the electron is separated from the nucleus and ionization occurs. When an electron drops down from a certain energy level to another lower level or to the ground state, energy corresponding to the drop is emitted in the form of *x-rays*. The wavelength of these x-rays depends upon the associated energy drop. The energy levels are measured in electron volts and excitation may be induced by electromagnetic influences. The nucleons within the nucleus behave in a similar manner. There are also discrete energy levels to which the nucleons can be excited. In dropping back to lower energy levels or to the ground state, the excess energy is emitted in the form of *γ-rays*. These energy levels are measured in mega-electron volts. Since the energy levels are roughly a million times greater than those for electrons, the excitation generally arises from particle interactions with the nucleus.

During the decay of radio-active isotopes, particles such as β-particles may be emitted. Since each such particle carries away a discrete amount of energy, it may leave the nucleus in an energy state higher than the ground state of the newly formed isotope. This excited isotope will then drop down to the new ground state by the emission of a γ-ray of appropriate energy. Invariably any nuclear reaction involving the absorption or emission of a particle by a nucleus results in the emission of a γ-ray.

2.1 Nuclear Structure

From the above it is apparent that the most stable nuclei, that is, those at the lowest energy levels have equal numbers of neutrons and protons. An excess of either neutrons or protons will push the nucleus to a higher energy level making it less stable. It would seem then that all elements should have equal numbers of neutrons and protons in their nuclei. This is not the case however.

If the structure of various atoms is studied it is found that, for the light elements, the number of neutrons and protons in the nucleus is about equal but, for the heavy elements, the number of neutrons exceeds the number of protons. If the number of neutrons in the nucleus is plotted against the number of protons for all stable isotopes of various elements, a graph that shows progressive departure from the line where the neutron number N equals the proton number Z, is obtained and is shown in Figure 1.

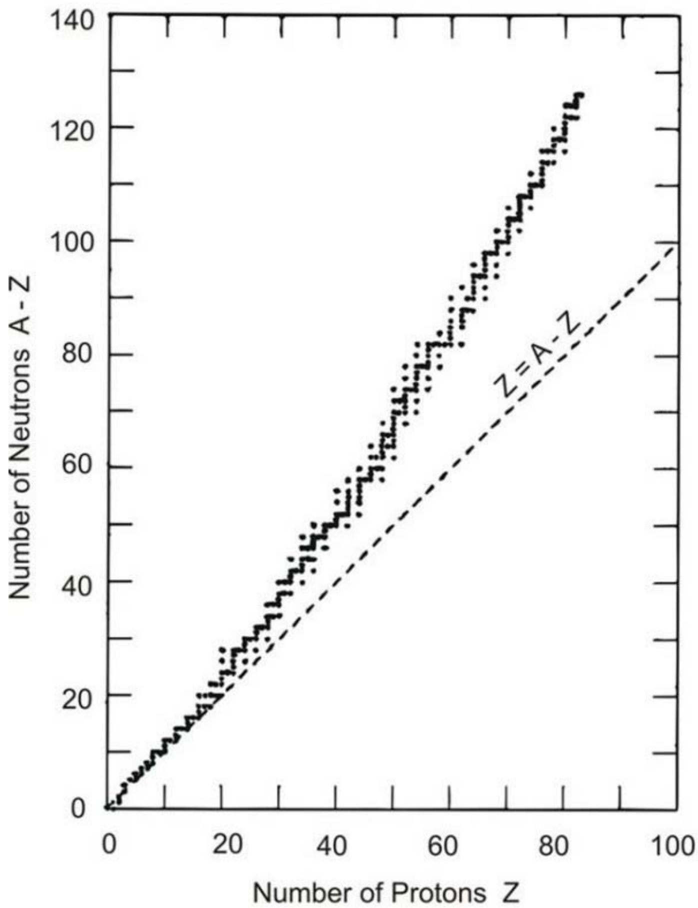

Figure 1: Neutron-proton ratio of stable elements

Within the nucleus there are *short range strong nuclear forces* and *long range electrostatic forces*. All nucleons attract one another due to the short range nuclear forces irrespective of type or charge but these forces only act on adjacent nucleons. The long range electrostatic forces however act over the entire nucleus causing repulsion between the similarly charged protons.

As the nucleus becomes larger with increasing atomic mass number, the influence of the long range electrostatic forces becomes greater since more positively charged protons are present. Under this influence the nucleus becomes unstable and, in order to hold it together, more neutrons are required. These additional neutrons help to bind the nucleus together with their short range nuclear forces and so dilute the effect of the long range electrostatic forces. At very high atomic mass numbers even these additional neutrons are not able to maintain a stable nucleus and all elements with an atomic number Z above 83 (Bismuth) are unstable.

Nuclei with too many neutrons or too few neutrons are also unstable. If there are too few neutrons, the excessive electrostatic forces of the protons create instability. If there are too many neutrons, the natural instability of the neutrons creates instability. In both cases there is a change in the nucleus to bring the isotope in question closer to the zone of stability. This zone is a curved band just above the $N = Z$ line on a plot of number of neutrons versus number of protons.

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Biographical Sketch

Robin Chaplin obtained a B.Sc. and M.Sc. in mechanical engineering from University of Cape Town in 1965 and 1968 respectively. Between these two periods of study he spent two years gaining experience in the operation and maintenance of coal fired power plants in South Africa. He subsequently spent a further year gaining experience on research and prototype nuclear reactors in South Africa and the United Kingdom and obtained M.Sc. in nuclear engineering from Imperial College of London University in 1971. On returning and taking up a position in the head office of Eskom he spent some twelve years initially in project management and then as head of steam turbine specialists. During this period he was involved with the construction of Ruacana Hydro Power Station in Namibia and Koeberg Nuclear Power Station in South Africa being responsible for the underground mechanical equipment and civil structures and for the mechanical balance-of-plant equipment at the respective plants. Continuing his interests in power plant modeling and simulation he obtained a Ph.D. in mechanical engineering from Queen's University in Canada in 1986 and was subsequently appointed as Chair in Power Plant Engineering at the University of New Brunswick. Here he teaches thermodynamics and fluid mechanics and specialized courses in nuclear and power plant engineering in the Department of Chemical Engineering. An important function is involvement in the plant operator and shift supervisor training programs at Point Lepreau Nuclear Generating Station. This includes the development of material and the teaching of courses in both nuclear and non-nuclear aspects of the program.